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# Tracer Dynamics in Two-layer Density-stratified Estuarine Flow

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**We consider tidal-driven particle motion in Tampa Bay, Florida under fully-stratified conditions with riverine inflow. The flow field is modelled by means of the two-layer shallow water equations solved on an adaptive quadtree grid using a second-order Roe-type finite volume scheme. Tracer dynamics are then obtained through Lagrangian particle tracking. The results show that the effect of external forcing is concentrated primarily in the upper layer, with particles in the lower layer less mixed and exhibiting relatively longer residence time in the semi-enclosed tidal bay. Particle flushing from Old Tampa Bay into the open sea is particularly hindered by the narrow inlet, whereas particles in Hillsborough Bay experience rapid transport seaward due to the large influx of river discharge. Particle patches are typically stretched in the Middle Tampa Bay and may be trapped in the shallow Lower Tampa Bay region. The study reveals interesting underlying mixing features that cannot be produced by a single-layer depth-averaged shallow water model.**

## Introduction

Density stratification is commonly found in open water bodies. Oceans and lakes have thermal stratification with an upper layer of water heated by solar radiation on top of denser, colder deep water. Stratification in estuaries is dominated by the effect of salinity difference. In general, stratified water bodies are characterised by a layered vertical structure with each layer having a distinct density. Under the influence of external forcing, such as wind shear, tidal cycle, and variable river runoff, the denser layer beneath may oscillate, resulting in cyclic rise and fall of the layer interface, and potential outcropping at the free surface. Consequently, tracers released from a fixed location near the domain boundary may be entrained into either the upper or lower layer. In the absence of vigorous vertical stirring, and noting that the flow dynamics arising from geographical and bathymetric features on the two layers may be very different, the tracers may thus be confined to horizontal dispersion in either layer and so are subjected to dissimilar fates. Although tracer dynamics of surface pollutants in environmental fluids has been widely pursued (Zimmerman, 1986; Budyansky *et al.*, 2007; Liang *et al.*, 2006a & 2006b; Pattantyús-Ábrahám *et al.*, 2008), there is a lack of investigations on the study of pollutants trapped below a stable interface in the lower layer of a density stratified flow.

In this paper, we consider tracer dynamics in a tidal-driven flow in an estuary, which is assumed to exhibit a pronounced two-layered density structure. The hydrodynamics can be adequately captured with a two-layer shallow flow model in the form of a hyperbolic system with non-conservative products and source terms. Although there may be no distinct separation interface between the layers, the two-layer approximation offers considerable computational advantages in resolving the fluid mechanical behaviour of such flows for practical engineering purposes. The numerical model is a second-order Roe-type finite volume scheme and is solved on a dynamically adaptive quadtree grid (Lee *et al.*, 2011). Lagrangian particle tracking is then performed at both the surface and lower layers to contrast the dynamics and eventual fate of tracers in either layer.

## Preliminaries

Tampa Bay (Figure 1) is located to the west of central Florida in the United States of America (USA). It comprises four sub-regions, namely Old Tampa Bay, Hillsborough Bay, and the Middle and Lower Tampa Bay. It is used for shipping, recreation, municipal fresh water consumption, sanitation requirements, and power generation. It is also the home of four aquatic reserves for several species of

marine life on both the state and federal lists of endangered species (Weisberg & Zheng, 2006). With such heavy and diverse usage, the importance of the ecological health of the estuary cannot be over-emphasized.



Figure 1. Tampa Bay watershed (Source: Florida's Water homepage)

By classification, Tampa Bay constitutes a drowned riverbed estuary with the mean low water depth increasing from about 4 m to 15 m along the dredged navigational channel. The bay length is about 50 km and has a maximum width of approximately 15 km at the midsection. The surface area of the estuary is approximately 1000 km<sup>2</sup>, and that of the surrounding watershed is approximately 4600 km<sup>2</sup>. Hydrodynamic advection in the bay is governed primarily by tidal flow from the adjoining Gulf of Mexico, and to a lesser degree by river inflow and wind. Tidal flow, which accounts for up to 95% of the current energy, largely determines the transport of chemical tracers, nutrients, organic and inorganic micro-pollutants. Mean annual river water discharge is about 63 m<sup>3</sup>/s. Despite being weaker by an order of magnitude compared to the tidal current, density-driven circulation remains an important mode of circulation (Weisberg & Zheng, 2006; Meyers & Luther, 2008).

### Governing equations

We consider a fully-stratified two-layer estuarine flow characterised by two superposed immiscible layers of shallow water fluids. The governing equations are given by a hyperbolic system with non-conservative products and source terms (Lee, 2011; Lee *et al.*, 2011):

$$\mathbf{W}_t + \mathbf{F}_1(\mathbf{W})_x + \mathbf{F}_2(\mathbf{W})_y = \mathbf{B}_1(\mathbf{W})\mathbf{W}_x + \mathbf{B}_2(\mathbf{W})\mathbf{W}_y + \mathbf{S}_1(x, \mathbf{W}) + \mathbf{S}_2(y, \mathbf{W}),$$

where the subscripts  $x$ ,  $y$  and  $t$  denote partial derivatives with respect to the  $x$ -direction,  $y$ -direction, and time.  $\mathbf{W}$  is the vector of unknowns;  $\mathbf{F}_1$  and  $\mathbf{F}_2$  are the flux function vectors;  $\mathbf{S}_1$  and  $\mathbf{S}_2$  are the source term vectors which describe the variable bed topography;  $\mathbf{B}_1$  and  $\mathbf{B}_2$  are the coupling matrices. The numerical solver comprises a second-order Roe-type finite volume scheme. The system is assumed to be strictly hyperbolic. This implies 6 distinct eigenvalues  $\lambda_i$ ,  $i = 1, 2, \dots, 6$  in each of the  $x$ - and  $y$ -directions respectively, with linearly independent eigenvectors. Two of the eigenvalues are equal to the layer-averaged flow speeds in the upper and lower layers, such that  $\lambda_j = U_j$  where  $j = 1, 2$ . Here, subscript  $j$  denotes the layers, where the indexes 1 and 2 refer to the upper layer and the lower layer respectively. The other eigenvalues are determined from the characteristic equation for the eigensystem:

$$(\lambda^2 - 2U_1\lambda + U_1^2 - gh_1)(\lambda^2 - 2U_2\lambda + U_2^2 - gh_2) = rg^2h_1h_2,$$

where  $h_j$  and  $\rho_j$  are the water depth and density of the respective layer, and  $g = 9.81\text{m/s}^2$ . For  $r \approx 1$ , the first order approximation of the eigenvalues can be written as (Schijf & Sconfeld 1953):

$$\lambda_{\text{ext}} = \frac{h_1 U_1 + h_2 U_2}{h_T} \pm \sqrt{g h_T}, \quad \lambda_{\text{int}} = \frac{h_1 U_2 + h_2 U_1}{h_T} \pm \sqrt{\frac{g' h_1 h_2}{h_T} \left( 1 - \frac{(U_2 - U_1)^2}{g' h_T} \right)}$$

where the total depth,  $h_T = h_1 + h_2$  and the reduced gravity,  $g' = g(1 - r)$ . The internal eigenvalues  $\lambda_{\text{int}}$  are generally much lower than the external eigenvalues  $\lambda_{\text{ext}}$ , each of which are related to the baroclinic and barotropic components of the flow respectively. For the system to remain stable, the approximate condition  $(U_2 - U_1)^2 / g' h_T < 1$  must be satisfied, implying a small velocity difference between the layers. Unless viscosity is introduced, instability could arise if this condition is not satisfied.

The bed stress components are calculated empirically from  $\tau_{bx} = \rho_j C_b u_j \sqrt{u_j^2 + v_j^2}$  and  $\tau_{by} = \rho_j C_b v_j \sqrt{u_j^2 + v_j^2}$  where the bed roughness coefficient is  $C_b = [\kappa / (1 + \ln(z_0 / h_j))]^2$  in which  $\kappa$  is the von Karman constant,  $z_0$  is the roughness height, and  $h_j$  is the instantaneous depth of the layer in contact with the bed. The layer-averaged eddy viscosity is given by  $\varepsilon = \kappa u_* h_j / 6$  where  $u_*$  is the bed friction velocity. The interfacial stress components are calculated from  $\tau_{fx} = \rho_2 C_f u_r \sqrt{u_r^2 + v_r^2}$  and  $\tau_{fy} = \rho_2 C_f v_r \sqrt{u_r^2 + v_r^2}$  where the Roe-averaged velocity changes are  $u_r = u_1 - u_2$  and  $v_r = v_1 - v_2$ , and  $C_f$  is the interface coefficient whose value was set arbitrarily to 0.03.

### Flow field simulation

For the purpose of the present study, we consider the effect attributed to the interactions of tidal forcing and riverine inflow only, and wind forcing has been omitted. Thirteen (13) river inflow locations are used, including Hillsborough (23.8%), Manatee (15.9%), Alafia (20.6%), Little Manatee (9.5%), with the balance contributed by the rest of the locations in proportion to their respective catchment size (Figure 2). The initial mesh is generated using an adaptive quadtree grid system (Figure 3). Maximum grid refinement is applied at the mouth of Hillsborough River, which represents the main inflow location and thus the localized high velocity gradient. This highest resolution is also used for the strait between Old Tampa Bay and Middle Tampa Bay, as well as at the wet-dry interface of the lower layer to circumvent interpolation at the transition between single and dual-layer flow. The corresponding maximum and minimum horizontal grid resolutions are thus 97.66 m and 781.25 m, respectively.

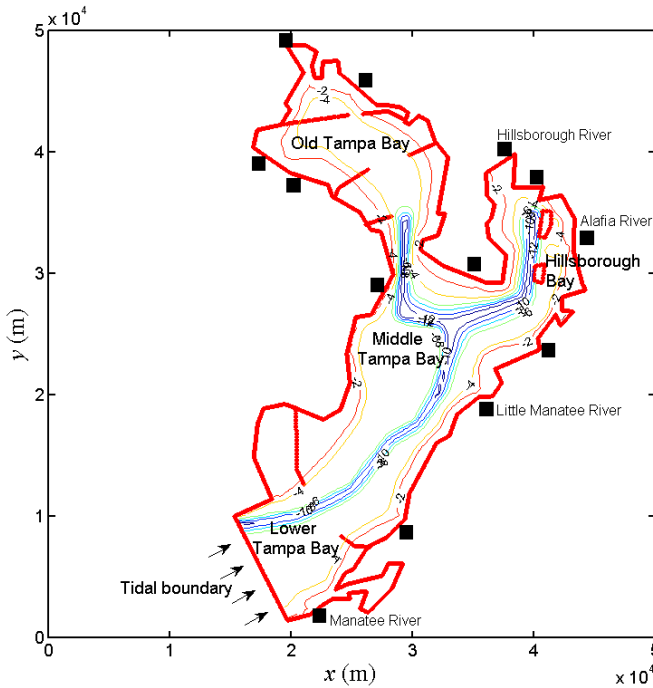


Figure 2. Computational domain, showing tidal boundary, river inlets (■), and discretised bathymetry

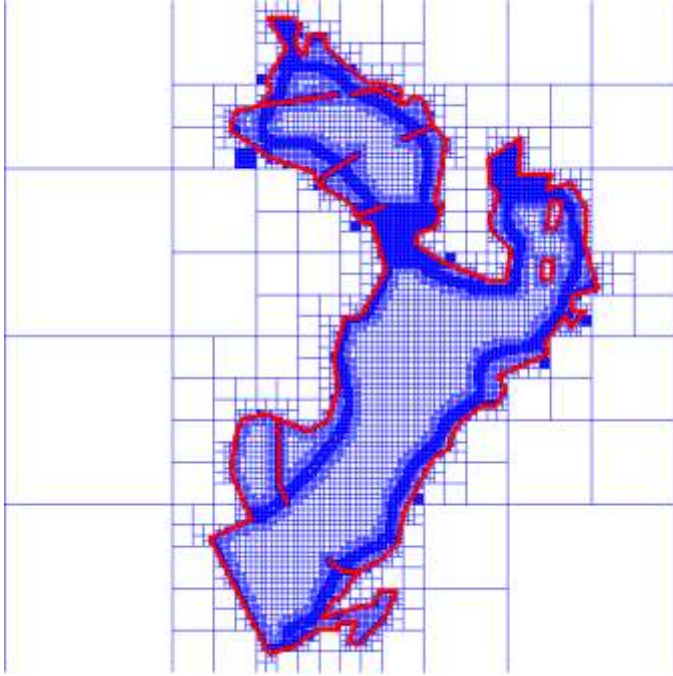


Figure 3. Computational grid. Plot showing refined mesh used in specific areas (e.g. in the vicinity of the Hillsborough inlet, and the narrow strait between Old Tampa bay and Middle Tampa Bay), and at the transition from two-layer to single layer fluid, typically near the domain boundary.

The densities of the upper and lower layers are taken to be  $1020.65 \text{ kg/m}^3$  and  $1025 \text{ kg/m}^3$ , respectively, i.e. density ratio  $r \sim 0.9958$ . Low riverine inflow equivalent to  $1.2 \times 10^6 \text{ m}^3/\text{day}$  (Meyers & Luther, 2008) is considered such that the initial thickness of the upper layer may be assumed constant at 2 m depth arbitrarily, and the initial sea water level corresponds to the mean sea level ( $z = 0$ ). These uniform inflows ( $\rho_1 = 1020.65 \text{ kg/m}^3$ ) are injected into the upper layer at open boundaries at selected grid cells corresponding to the river discharge locations which are typically less than 2 m deep. A diurnal tide with tidal range of 1 m is imposed at the tidal boundary located at the southwest of the domain.

Simulated depth-averaged streamlines during flood and ebb tide in the bay over a typical 24-h cycle for low riverine inflow condition verify that the circulation is tidal-dominant (Figure 4). The average velocity in the surface layer is 2.9 cm/s which is comparable to that reported by Weisberg & Zheng (2006). The velocity in the lower, denser layer is smaller by up to two orders of magnitude than that of the surface layer for the present low inflow case considered. Hence, tidal energy is essentially concentrated in the upper layer whereas hardly any flow takes place in the layer beneath. For the grid scale chosen, no distinct recirculation zone is identified, despite the presence of island structures and groynes in the domain. Flow features observed in the two layers are identical, largely due to the dominant effect of tidal flushing. The relatively low flow velocity implies there is little or no vertical convection between the layers, and so the two-layer immiscible model is a reasonable approximation for the problem.

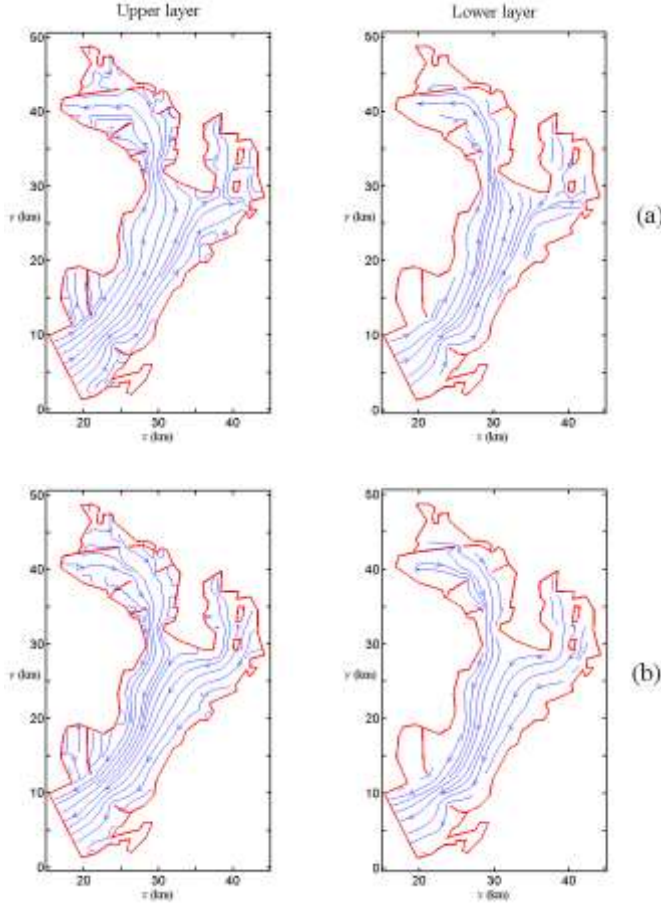


Figure 4. Depth-averaged streamlines during (a) flood tide, and (b) ebb tide.

### Tracer dynamics in two-layer stratified flow

For the purpose of Lagrangian particle tracking, time-integration of the advection equations is performed using a Runge-Kutta Cash-Karp algorithm on the interpolated Eulerian velocity field obtained from the two-layer shallow water model described in the preceding section. A tidal cycle corresponding to 24 hour period is typified using ten (10) flow fields extracted at equal intervals, which are then looped to imitate cyclic flood and ebb conditions. Essentially, a simple time-periodic flow is produced which is responsible for the advection of the passive particles in both layers. Diffusion is ignored, given the relatively large scale of the shallow water flow domain.

The advection equation in a two-dimensional Cartesian coordinate system is expressed as

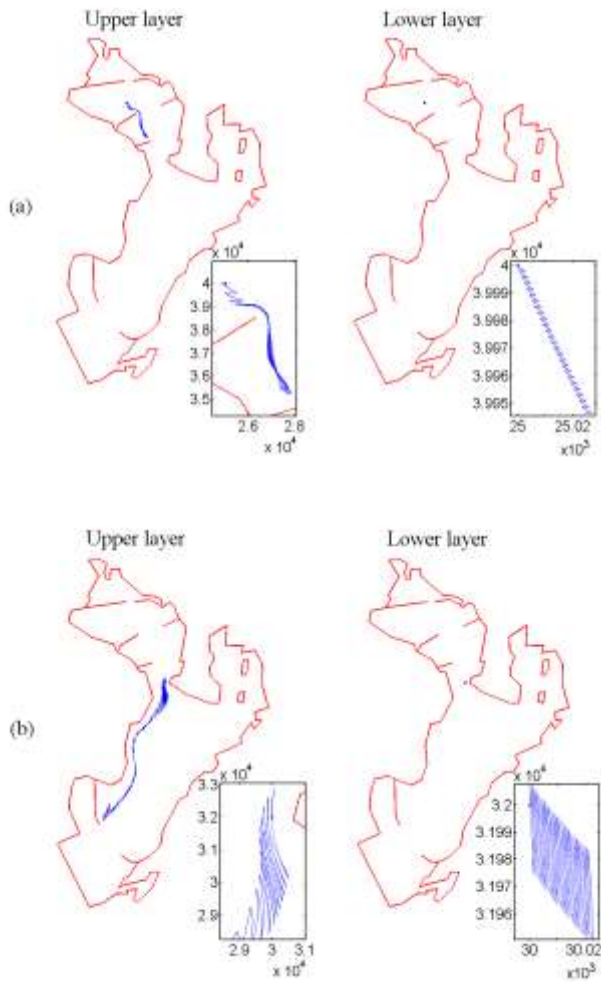
$$\frac{dx}{dt} = u(x, y, t), \quad \frac{dy}{dt} = v(x, y, t)$$

where  $(x, y)$  is the position of a given particle with respect to an arbitrary origin at time  $t$ ; and  $u$  and  $v$  denote the Eulerian velocity components in the  $x$ - and  $y$ -directions of the flow at the same spatial and temporal point as the particle (see e.g. Liang *et al.*, 2006a,b). The required continuous velocity field is interpolated linearly across the discretised representation available from adjacent cells of the computational grid.

For the study of particle advection, a set of particles is deployed at the same initial positions in both layers at selected locations, and the particles tracked for up to 25 cycles. Figure 5 shows the trajectories of the particles in both the upper and lower layers for 4 different initial positions (a-d). In all cases considered, although particles in the upper layer are advected a substantial distance after 25 cycles, advection in the lower layer is almost negligible owing to the large velocity difference between the layers. There is, however, a clear net drift out of the bay in both layers. Considering the particles in the upper layer, the results show that particles in the shallow Old Tampa



Bay have relatively low velocities and are just about to exit into the Middle Tampa Bay after 25 cycles (Figure 5a(i)). Once a particle reaches the mouth of Old Tampa Bay, it moves at a faster rate alongshore in the deeper trench to the west of the shallow central rise region in Middle Tampa Bay (Figure 5b(i)). A particle seeded in Hillsborough Bay (Figure 5c(i)), on the other hand, advects relatively rapidly along the main axis of Tampa Bay, which is to the east of the central rise in the Middle Tampa Bay region where the deep navigational channel is located. This advection route can be attributed to the relatively large inflow from the Alafia River, which accounts for one-fifth the river inflow in Tampa Bay. A particle seeded near the upstream end of the Lower Tampa Bay is flushed out of the basin relatively quickly (Figure 5d(i)), and hence this region is not of interest in the subsequent study.



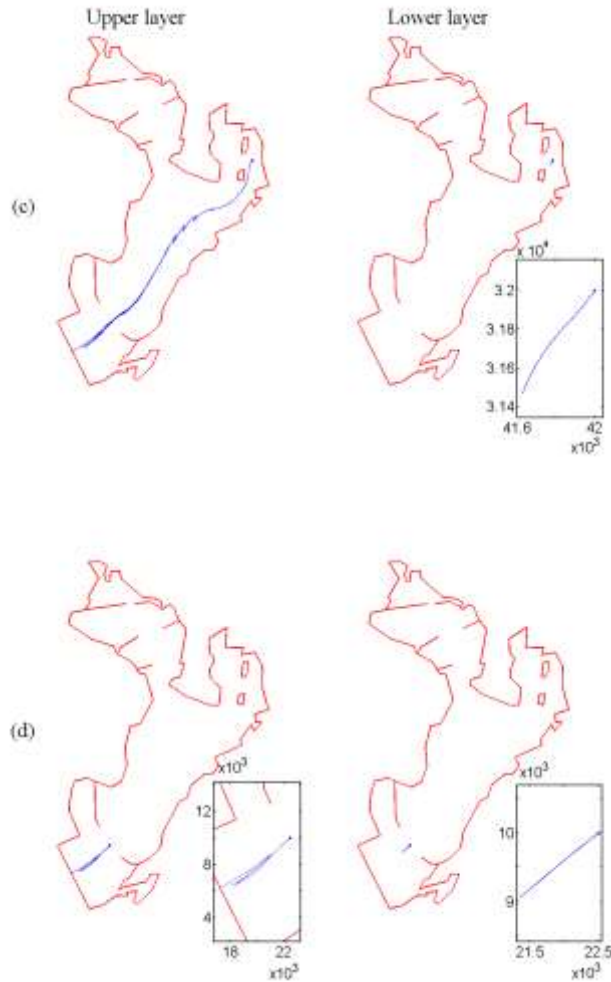


Figure 5. Trajectories of a single particle for a period of 25 cycles. Particle initial positions (marked  $\bullet$ ) are: (a)  $(2.5 \times 10^4, 4.0 \times 10^4)$ , (b)  $(3.0 \times 10^4, 3.2 \times 10^4)$ , (c)  $(4.2 \times 10^4, 3.2 \times 10^4)$ , and (d)  $(2.25 \times 10^4, 1.0 \times 10^4)$ . Insets show details of the respective trajectories.

Next, a square patch of 10,000 particles is deployed in each layer at different initial locations. Figure 6a(i) shows that by 100 cycles, the square patch from the Old Tampa Bay region is greatly stretched along the eastern shore of Middle Tampa Bay but a large number of particles still remain at the mouth of Old Tampa Bay. The narrow strait connecting the Old and Middle Tampa Bay acts like an hourglass neck (the Bernoulli effect) and releases particles from the Old Tampa Bay in a constrained manner. This results from the flood tide which forces some of the escaping particles back into the Old Tampa Bay before they are entrained into the alongshore coastal jet in the Middle Tampa Bay. Further evidence is provided by Figure 5b(i) where the trajectory of a single particle near the mouth of Old Tampa Bay undergoes cyclic to-and-fro motions before it is finally washed out of the strait region. Figure 6a(i) also shows that the particles become increasingly scattered as they approach Lower Tampa Bay, and spread out in the shallow lagoon in the vicinity of the groyne. The results show that 65.2% of particles still remain in Old Tampa Bay after 100 cycles, and this percentage reduces gradually to 48.9% at the end of 200 cycles, suggesting that the strait is an efficient barrier to the outflow of tracers. Furthermore, the particles that escape Old Tampa Bay are mostly scattered in the shallow lagoon near Lower Tampa Bay, and only 11.3% exit the domain by the end of 200 cycles. Our observations are in line with the arguments of Weisberg & Zheng (2006) on gravitational convection in the bay under two-layered structure where the journeys of particles at depth are more hindered by the bed variations whereas the Lagrangian pathway near the surface assumes relatively straightforward routes to the bay exit with residence times of up to 100 days.



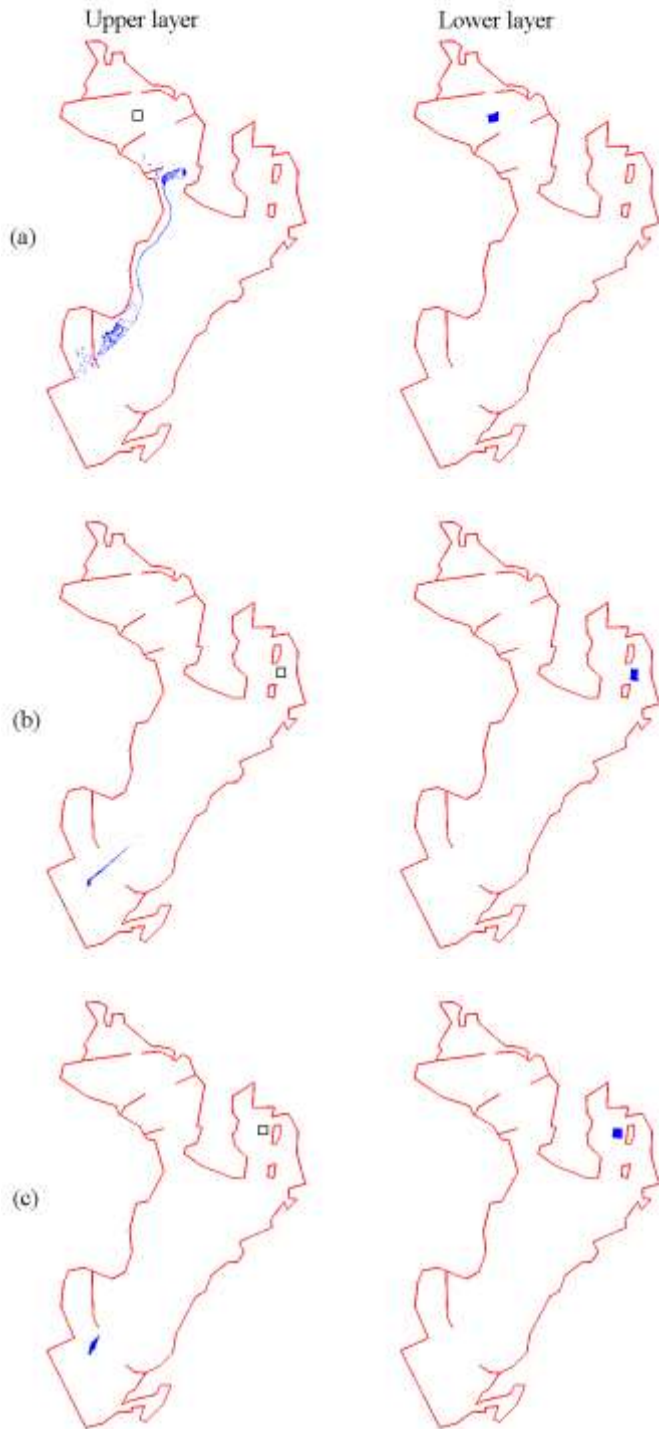


Figure 6. Advection of a square patch of particles. Plots showing particle positions after 100 cycles (a), and 15 cycles (b,c). Initial particle positions (marked  $\square$  on the upper layer) is centered at: (a)  $(2.55 \times 10^4, 4.05 \times 10^4)$ , (b)  $(4.15 \times 10^4, 3.2 \times 10^4)$ , and (c)  $(3.95 \times 10^4, 3.45 \times 10^4)$ .

Two square patches of particles are deployed in Hillsborough Bay. The patch near the mouth of River Alafia (Figure 6b(i)) ( $B_1$ ) is significantly stretched but not scattered as it is advected towards the Lower Tampa Bay. The other patch is located initially in the relatively shielded region to the west of the northern island in Hillsborough Bay (see Figure 6c(i)) ( $B_2$ ). This initial location is away from

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the direct river influxes especially the inflow from the Hillsborough River, which accounts for almost a quarter of the river inflow in Tampa Bay. Interestingly, whereas  $B_2$  is advected at almost the same rate as  $B_1$ , it clearly experiences much less stretching. The two square patches of particles in the Hillsborough Bay,  $B_1$  and  $B_2$  exit the computational domain within 18.4 cycles and 16.3 cycles, respectively. In the former case, the longer duration taken is due to the stretching of the particle patch. It is also worth noting that close to 7% of the particles in  $B_1$  are washed ashore on the southern island in the Hillsborough Bay and are not further entrained into the flow field.

## Discussions & Conclusions

The energy of the two-layer stratified Tampa Bay under low river inflow condition is concentrated in the upper surface layer and can be attributed almost entirely to tidal forcing, with a net drift out of the bay. Particle advection and mixing in the denser fluid below is extremely poor with excessively long residence time, with important implications for water quality in the lower layer. Particles in the surface layer, on the other hand, undergo rather different rates of advection and mixing dependent on their location. This is because the surface layer typically experiences a relatively stronger ebb flow compared to the flood flow over a tidal cycle due to the effect of the river input. The opposite is true for the lower or subsurface layer. Hence, the difference between streamwise particle advection in the upper and lower layers is likely to increase for large riverine inflow. Old Tampa Bay has a relatively long residence time due to the narrow outlet, whereas Hillsborough Bay has a relatively short residence time due to the large fluxes of inflow from the tributaries. Middle Tampa Bay is where particles are typically stretched and washed out. Particles which are not scattered into the slow-flowing shallow lagoon adjacent to the Lower Tampa Bay are rapidly flushed out of the domain.

Whilst this paper describes a purely hydrodynamic model for the mixing behaviour of two-layer water bodies, the model could be coupled with reaction diffusion type equations to make potentially useful predictions of changes of, for example, dissolved oxygen levels or more general biological activity in lakes. Such work could use the methodology of advancing fronts developed by Károlyi *et al.* (1999).

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